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IMPROVED ELEMENT-FREE GALERKIN METHOD FOR ELECTROMAGNETIC NDE MODEL (PREPRINT)

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14. ABSTRACT This paper presents improvements made to enhance the Element-Free Galerkin method (EFG) for NDE applications. By using orthogonal basis functions instead of the conventional polynomial basis, the inversion of the ill-conditioned shape function matrix is eliminated. Therefore, the accuracy of the results is improved. Preliminary one-dimensional (1-D) and two-dimensional (2-D) examples are presented to demonstrate the improvement in accuracy and efficiency offered by orthogonal basis functions.					
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Improved Element-Free Galerkin Method for Electromagnetic NDE Model

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Abstract. This paper presents an improvement of the new Element-Free Galerkin method (EFG) in NDE application. By using the higher order orthogonal basis function instead of the conventional polynomial basis, the ill-condition and inversion of the shape function are eliminated. Therefore, the accuracy of the results is improved. The preliminary one-dimensional (1-D) and two-dimensional (2-D) examples are presented to demonstrate the accuracy and efficiency of the improvement.

Keywords. NDE, meshless method, EFG method, orthogonal basis

Introduction

Nondestructive Evaluation (NDE) methods are widely used in industry to control and maintain product qualities. The availability of a numerical model is extremely important to visualize the field-flaw interaction in NDE. Recently, meshless methods, known as a new numerical approach, have been developed to eliminate the disadvantage of the conventional Finite Element Method (FEM). Without the reliance of a mesh, meshless methods can be implemented to model complex geometries, such as tight crack and dynamic corrosions.

Element-Free Galerkin method (EFG) is one of meshless methods by utilizing moving least square (MLS) approximation and Galerkin formulation. The MLS relies on the following three components: a weight function, a basis function, and a set of position-dependent coefficients.

We discovered that the shape function is ill-conditioned when using high order polynomial basis. This will prevent the implementation of high order polynomial basis for higher accuracy. To solve this problem, an improved formulation of the EFG method using orthogonal basis function is proposed in this paper. The condition of the shape function is improved compared with the polynomial basis and the convergence of the iteration solver is faster and the accuracy of the solution is increased.

1. Formulations

In the EFG method, a set of nodes are used to construct the discrete system of equations approximating the solution. And a set of quadrature points are defined to compute the integrals over the solution domain in Galerkin procedure.

The MLS approximation can be written as

$$u^h(\mathbf{x}) = \sum_{j=1}^n \Phi_j(\mathbf{x}) u_j \quad (1)$$

where n is the total number of nodes and the shape functions Φ_j are given by

$$\Phi_j(\mathbf{x}) = \sum_{i=0}^m p_i(\mathbf{x}) (\mathbf{B}^{-1}(\mathbf{x}) \mathbf{C}(\mathbf{x}))_{ij} = \mathbf{p}^T \mathbf{B}^{-1} \mathbf{C}_j. \quad (2)$$

where $\mathbf{B}(\mathbf{x}) = \mathbf{P}^T \mathbf{W}(\mathbf{x}) \mathbf{P}$, $\mathbf{C}(\mathbf{x}) = \mathbf{P}^T \mathbf{W}(\mathbf{x})$, \mathbf{W} is the weight function, \mathbf{P} is the basis function and m is the order of the basis function.

The choice of basis function is essential for the stability of the numerical results because there is an inversion of matrix \mathbf{B} . The ordinary polynomial basis function is straight for implementation but results in an ill-condition for high-order, which makes the solution not convergent.

However, an orthogonal basis can be used to solve this problem. The definition and formulations of the orthogonal basis are shown below:

Vector $\mathbf{p}(\mathbf{x}) = [p_1(\mathbf{x}), \dots, p_j(\mathbf{x}), \dots, p_m(\mathbf{x})]^T$ is an orthogonal basis, if we define the inner product as

$$\langle p_l(\mathbf{x}), p_k(\mathbf{x}) \rangle = \sum_{i=1}^N w(\mathbf{x} - \mathbf{x}_i) p_l(\mathbf{x}_i) p_k(\mathbf{x}_i) = \begin{cases} 0 & (l \neq k) \\ C_l \neq 0 & (l = k) \end{cases} \quad (3)$$

Therefore, its shape matrix becomes

$$\mathbf{B}(\mathbf{x}) = \begin{bmatrix} \langle p_1(\mathbf{x}), p_1(\mathbf{x}) \rangle & & \\ & \ddots & \\ & & \langle p_m(\mathbf{x}), p_m(\mathbf{x}) \rangle \end{bmatrix} \quad (4)$$

The recursive relationship of orthogonal basis is given by,

$$\begin{cases} p_1(\mathbf{x}) = 1 \\ p_2(\mathbf{x}) = (\mathbf{x} - \alpha_2) p_1(\mathbf{x}) \\ p_{k+1}(\mathbf{x}) = (\mathbf{x} - \alpha_{k+1}) p_k(\mathbf{x}) - \beta_k p_{k-1}(\mathbf{x}) \end{cases} \quad (5)$$

where $k = 1, 2, \dots, m$. The parameters α, β are obtained as,

$$\begin{cases} \alpha_{k+1} = \frac{\langle \mathbf{x} p_k(\mathbf{x}), p_k(\mathbf{x}) \rangle}{\langle p_k(\mathbf{x}), p_k(\mathbf{x}) \rangle} \\ \beta_k = \frac{\langle p_k(\mathbf{x}), p_k(\mathbf{x}) \rangle}{\langle p_{k-1}(\mathbf{x}), p_{k-1}(\mathbf{x}) \rangle} \end{cases} \quad (6)$$

where $k = 0, 1, \dots, m-1$. Substituting (4)-(6) into (2), the shape functions are then expressed as,

$$\Phi_i(\mathbf{x}) = \sum_{j=1}^m \frac{w(\mathbf{x} - \mathbf{x}_i) p_j(\mathbf{x}_i)}{\langle p_j(\mathbf{x}), p_j(\mathbf{x}) \rangle} p_j(\mathbf{x}) \quad (7)$$

The partial differential form of the shape function is

$$\begin{aligned} \frac{d\Phi_i(\mathbf{x})}{d\mathbf{x}} &= \sum_{j=1}^m \frac{\frac{dw(\mathbf{x} - \mathbf{x}_i)}{d\mathbf{x}} p_j(\mathbf{x}_i) p_j(\mathbf{x})}{\langle p_j(\mathbf{x}), p_j(\mathbf{x}) \rangle} + \sum_{j=1}^m \frac{w(\mathbf{x} - \mathbf{x}_i) p_j(\mathbf{x}_i) \frac{dp_j(\mathbf{x})}{d\mathbf{x}}}{\langle p_j(\mathbf{x}), p_j(\mathbf{x}) \rangle} \\ &\quad - \sum_{j=1}^m \frac{w(\mathbf{x} - \mathbf{x}_i) p_j(\mathbf{x}_i) p_j(\mathbf{x}) \frac{d\langle p_j(\mathbf{x}), p_j(\mathbf{x}) \rangle}{d\mathbf{x}}}{\langle p_j(\mathbf{x}), p_j(\mathbf{x}) \rangle^2} \end{aligned} \quad (8)$$

And the partial differential of inner product form is

$$\frac{d\langle p_j(\mathbf{x}), p_j(\mathbf{x}) \rangle}{d\mathbf{x}} = \left[\sum_{i=1}^N w(\mathbf{x} - \mathbf{x}_i) [p_j(\mathbf{x}_i)]^2 \right] = \sum_{i=1}^N \frac{dw(\mathbf{x} - \mathbf{x}_i)}{d\mathbf{x}} [p_j(\mathbf{x}_i)]^2 \quad (9)$$

These equations (3)-(9) indicate that the shape functions in EFG method avoid the inversion of shape function matrix when orthogonal basis is used. This method makes the implementation of EFG method easier, also increases the accuracy of the solution by improving order of the orthogonal basis.

2. Implementation

2.1 One-Dimensional Electrostatic Problem

A 1-D electrostatic Poisson problem for the potential function u is described as follows:

$$\begin{cases} \nabla^2 u = -\mathbf{x} & (\Omega : 0 < \mathbf{x} < 1) \\ \mathbf{x}(0) = 0 & u'(1) = 0 \end{cases} \quad (10)$$

EFG method is implemented with polynomial basis and orthogonal basis of various orders for comparison. The exact solution is $u(\mathbf{x}) = \frac{1}{2}\mathbf{x} - \frac{1}{6}\mathbf{x}^3$ for comparison with the numerical solutions.

Fig. 1 shows the exact solution and the numerical solution using EFG method with 1st and 2nd order polynomial basis. We discovered that, although the solution with 2nd order polynomial basis convergences, it starts to show unstable.

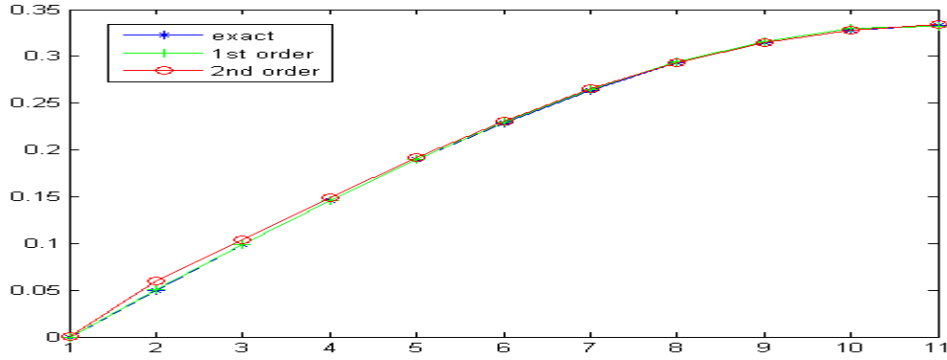


Figure 1. The comparison of exact and EFG solution with 1st, 2nd order polynomial basis.

Fig. 2 shows the exact solution as well as the solution using EFG method with 3rd order polynomial basis. The numerical solution is no longer convergent because of the ill-conditioned shape function.

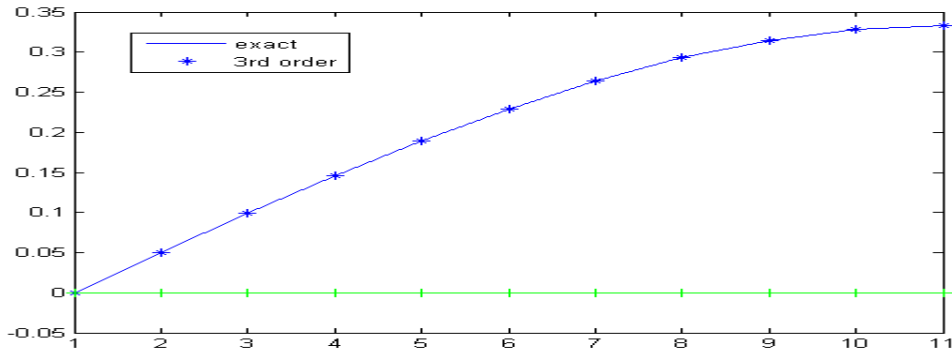


Figure 2. The comparison of exact and EFG solution with 3rd order polynomial basis.

In Fig. 3, the numerical solution using EFG method with orthogonal basis function is presented. The order of the basis function varies from 1st to 4th; the numerical solutions are stable and keep convergent, which validate the formulation of the orthogonal basis function.

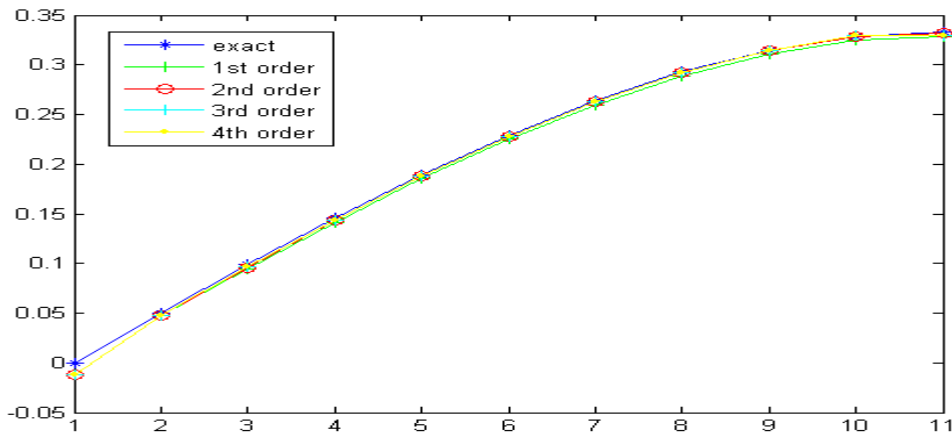


Figure 3. The comparison of exact solution with various order orthogonal basis.

2.2 Two-Dimensional Eddy Current Problem

The 2-D eddy current modeling geometry is shown in Fig. 4. Assume an infinite-size 2-D conducting plate with a tiny crack along y direction (length = 1) is placed under a time-varying harmonic current source in the x direction.

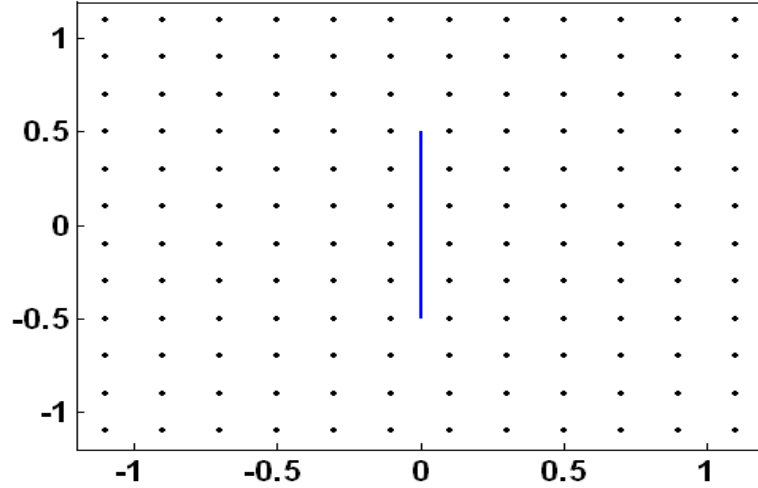


Figure 4. Modeling geometry for 2-D problem.

The induced current will be along $-x$ direction in the absence of crack, and will be bended by its presence. This geometry is a typical 2-D eddy current problem and the plot of the induced eddy current is shown below.

Fig. 5 (a) shows the induced eddy current using EFG method by the 1st order polynomial basis function with the existence of the crack and (b) shows the image using EFG method by the 2nd order polynomial basis function. Due to the complexity of the problem, the solution with the 2nd order polynomial basis function is no longer convergent. And there is no need to consider much higher-order basis function.

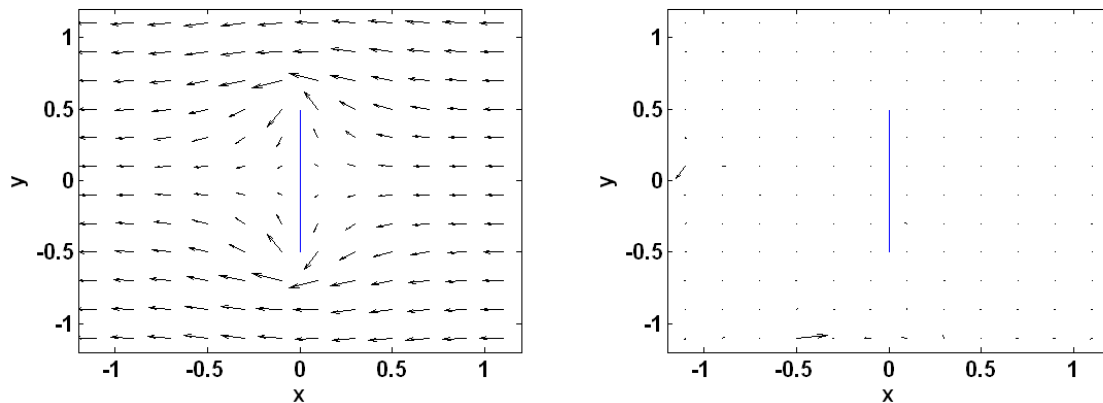


Figure 5. Induced eddy current plot using EFG method by polynomial basis function (a) 1st order (b) 2nd order.

Fig. 6 (a) shows the current plot using EFG method by 2nd order orthogonal basis and (b) shows the difference compared with Fig. 5 (a). It is very clear that the 2nd order orthogonal basis function still converges and only minor difference is observed compared with the 1st order polynomial basis function.

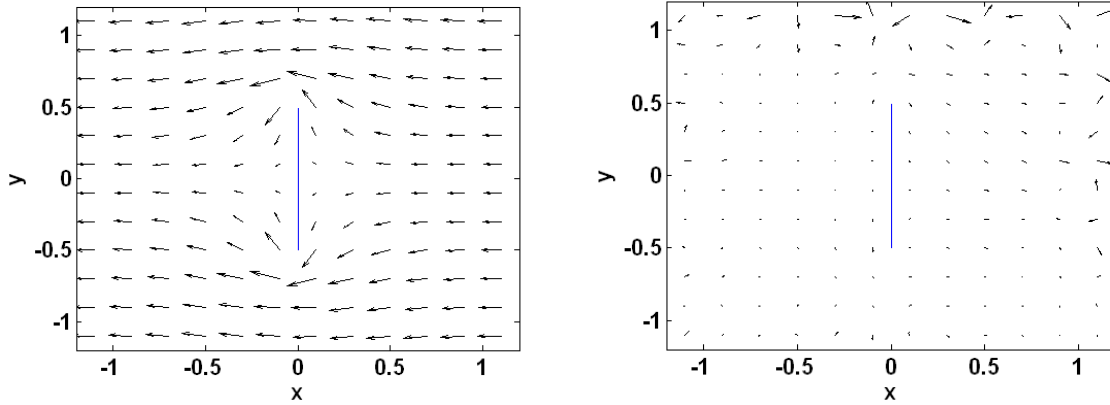


Figure 6. Induced eddy current plot using EFG method by orthogonal basis function (a) 2nd order (b) comparison with Fig. 5 (a).

When using orthogonal basis function, there is no need to perform the LU decomposition because of the avoidance of the shape function inversion. Fig. 7 shows the relative iteration error during convergence for 1st order polynomial basis and 2nd order orthogonal basis. We can conclude that the usage of the orthogonal basis function will result in a faster convergence.

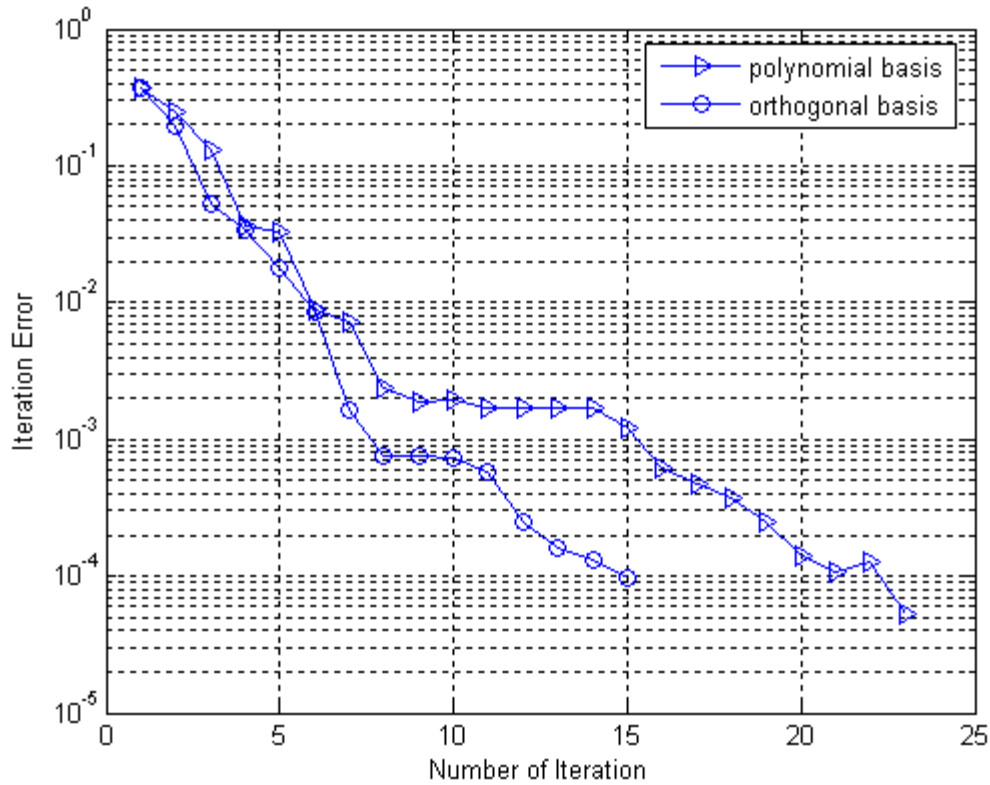


Figure 7. the iteration error versus iteration number for polynomial basis and orthogonal basis.

3. Conclusion

Conclusion: from the comparison of the simulation results in 1-D and 2-D electromagnetics problems, the improved formulation of the EFG method is validated. The implementation of orthogonal basis function will avoid the inversion of shape function matrix and increase the stability of the system. It provides an opportunity for the application of higher-order basis, which will increase the solution accuracy consequently. The above 1-D and 2-D examples have already shown this advantage and the 3-D problem will potentially be simulated.